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Can Atomic Processes be Described by Non-linear Wave Mechanics in Space-time?

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Abstract

Generalizing from a classical application of the paradigm of Elementary Measurement discussed elsewhere (Leiter, 1969), we consider a non-linear, spinor wave-mechanical field theory of Elementary Measurement. In this theory, charged particles are represented by complex spinor c-number fields interacting through their associated electromagnetic fields in space-time. The paradigm of Elementary Measurement implies that the particle fields, and their associated c-number electromagnetic fields, are interdependent degrees of freedom in an action principle associated with the measurement interaction, and are not elementary in themselves. Making the action stationary with respect to the interacting field degrees of freedom gives the equations of motion of the measurement.

The application of this model theory to atomic hydrogen yields the result that the inherent 'limit cycle solutions' (LCS)‡ of the non-linear measurement equations correspond to the quantum levels of conventional relativistic Dirac quantum mechanics of hydrogen, in the approximation that the nucleus has infinite mass.§

Superpositions of these Dirac–LCS solutions have the property of collapsing (reduction of the wave packet) into one of the LCS in the superposition, *in a characteristic time which is identical to the 'lifetime' of the associated atomic levels as calculated from conventional quantum mechanics*. Hence, in this c-number electromagnetic theory, *both* spontaneous and induced transitions can be accounted for. 'Photons', in this theory, are not elementary particles, but instead are associated with the secondary dynamics related to the inherent nonlinear structure in the elementary measurement equations of motion. The 'hidden variable' characteristics of this measurement theory (as seen from the point of view of ordinary quantum mechanics), in describing a universe made up of such hydrogen atoms, is discussed. Within this context, a consistent derivation of the Planck blackbody radiation formula is given, in which the associated electromagnetic fields are c-numbers and are *not* second quantized. Finally, a generalization of this prototype model theory, to a more consistent form which can account for the presence of 'vacuum interaction processes' and negative energy states, is suggested.

† Preliminary study for this work was done at the Albert Einstein Institute of Physics, Technion, in Haifa, Israel, during the summer of 1965.

‡ Minorsky, N. (1962). *Nonlinear Oscillations*, Chaps. 2, 6 and 21 and pp. 73–74. Van Nostrand. Additional information on stability, albeit on an equation different from ours, is found in Echaus, V. (1965). *Studies in Nonlinear Stability Theory*. Springer-Verlag.

§ This means that the reduced mass correction is neglected in this model theory, but can be included by making the proper generalizations. In this regard, see footnote on page 218.

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1. *Introduction*

Neils Bohr (1958, 1963), in his later books on quantum theory, emphasized the fact that physical events *always* involve an observer as well as an object to be observed: '...any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly an independent reality in the ordinary physical sense can neither be ascribed to the phenomena or to the agency of observation'. Bohr qualifies this statement, in his 'Copenhagen Interpretation' of the measurement process, by pointing out that the measurement process cannot be understood as a causal interaction, between two mechanical systems, in any ordinary sense; '...the objects measured in a quantum measurement process have a different status in the theory to the measurement instruments...in that they belong to different levels of functioning which are not related mechanically'.

Basic difficulties arise when one tries to *operationally* define the concept of 'elementary particle' within the above framework, because the very act of trying to observe such 'elementary' entities involves interactions with other such entities in the measuring apparatus. In particular, the physical act of observing an 'elementary charged particle' always involves interactions with other such 'elementary' charged particles, because of the empirical existence of an indivisible, non-zero, unit of electric charge. Hence the concept of an elementary charged particle is basically non-operational, since the very act of observing such an entity destroys its 'elementarity'.

Because of the value of eliminating non-operational concepts from physical theories, one is led to eliminate the concept of 'elementary particle' from measurement theory, but cannot do so without changing the basic ideas upon which it is formed. Hence one is led to attempt to replace the conventional measurement paradigm, described above, by a new paradigm which gives a complete, operational description of the measurement process, without the concept of elementary particle. The new paradigm states that; *in a physical event, it is the 'mutual measurement interaction' between the 'observer' and the 'object to be observed' which is the 'elementary' building block of any theory attempting to describe that physical event.*†

This new paradigm, of the measurement process, is similar to that of Bohr, in that it implies that the measurement interaction can never be neglected in the observation of atomic processes. However, it differs from Bohr's approach in that the 'object to be measured' and the 'measuring instrument' are put on equal footing. This means that they belong to the same level of functioning (being related by the dynamics of the 'elementary

† At this point, the author wishes to make an important distinction between the work presented in this paper (and quoted in Leiter, 1969) and that of Sachs, M. and Schwebel, S. (1961). *Nuovo cimento*, **21**, 197, Suppl. No. 2. Specifically, the author wishes to point out that his work is related to the work of Sachs and Schwebel only in the fact that a similar basic paradigm is used.

measurement' in which they participate). This means that the 'elementary measurement' must be described in *space-time*, since that is the region in which it is physically observed.

Within the context of a classical electrodynamic theory of 'Elementary Measurement', charged point particles (associated with $J_\mu^{(K)}(\mathbf{x},t)$; $K=1, 2,..,N$) and their related electromagnetic fields, $A_\mu^{(K)}(\mathbf{x},t]$, are not elementary in themselves, but rather the mutual measurement interactions $J_\mu^{(K)}A_\mu^{(J)}$; $(K\neq J]$ are the fundamental building blocks of the theory. The resulting theory has been shown to be equivalent to that of the 'action-at-a-distance' electrodynamics of Wheeler & Feynman (1945, 1949),† with the distinct advantage that retardation and radiation reaction are accounted for, in a Lagrangian formalism, without the use of any 'complete absorber assumption'. This means that a theory such as this can account for all the physical phenomena associated with the Maxwell–Lorentz electrodynamics of point charges, without the paradoxical infinities associated with 'self-interaction'.

Because of the success of this paradigm in the classical context, the most natural next step is to develop a 'wave-mechanical' generalization of it. Hence what we propose to do, in this paper, is to develop a model non-linear Dirac wave-mechanical theory of 'elementary measurement' electrodynamics for 'hydrogen atom' measurement interactions.

Generalizing from the classical application of this paradigm discussed above, we shall develop a spinor model, non-linear wave-mechanical field theory of elementary measurement, in which charged 'particles' are not basically corpuscular in nature, but instead are represented (in space-time) by complex, spinor matter fields. These matter fields and their associated electromagnetic potential fields are not elementary in themselves, but are interdependent degrees of freedom in a scalar 'elementary measurement' field in space-time. This elementary measurement field structure represents the elements of the theory which are directly connected to the objective reality of 'observable physical phenomena'.

There are two layers to the structure of this theory: an abstract mathematical layer, and an objective physical layer. The former involves the spinor matter fields and their electromagnetic fields in space-time, while the latter involves the mutual interaction energies, momenta, etc. as derived from the 'elementary measurement' energy-momentum tensor, derived from the scalar elementary measurement field structure in the action principle. The matter fields and their electromagnetic fields are, by themselves, merely mathematical abstractions. They are accessible to observation, only by virtue of the mutual measurement interaction energies they can combine to produce. It is only these 'elementary measurement fields' which are objective physical quantities, directly accessible to physical observation.

In the following we shall discuss the application of this theory to a

† See also Leiter (1969).

relativistic description of 'hydrogen atoms', in the approximation that the proton nucleus has infinite mass. At this stage of the theory, 'vacuum and annihilation' effects, associated with the existence of negative energy states, are neglected. These effects, and also the Pauli Exclusion Principle, can be included in a more general version of the theory.†

There are no corpuscular 'elementary particles' in this theory, only charged 'matter waves' participating in 'elementary' electromagnetic measurement interactions in space-time. Hence the conventional 'wave-particle' duality does not arise in this type of measurement theory. From the Schwartz Inequality,‡ inherent in this theory, a mathematical equivalent of the Heisenberg Uncertainty Principle may be deduced, which implies the same physical consequences as that of the uncertainty principle in conventional quantum mechanics. However, the lack of wave-particle duality implies that 'probability' and 'uncertainty' are not fundamental to its interpretation.

The fact that the non-linear measurement field equations have 'limit cycle solutions' (LCS) localized into a spatial region on the order of 10^{-8} cm, will nonetheless give the atomic measurement process the *apparent* smeared out 'corpuscular' character usually associated with the conventional quantum uncertainty interpretation, but without any 'elementary corpuscles' actually being involved.

The LCS play the role of 'quantum levels' in this theory, and, even though they are the solution to an inherently non-linear differential equation, form a complete set of functions for the discreet part of the non-linear function space involved. For this reason, an arbitrary non-linear solution, spanning the discreet function space of the non-linear equations, can be represented by a superposition of LCS with time-dependent coefficients. We will show that these superpositions of LCS have the inherent property of collapsing continuously, in a sequence which depends on the 'selection rules' involved, into the LCS contained in the superposition. Hence, in this theory, these solutions [which belong to the class of 'limit cycle transition' solutions (LCT)] will give a description of 'quantum jumps' in terms of a continuous space-time picture. The (LCT) solutions have the property of evolving from one LCS to another in a characteristic time similar to that of the 'lifetime' which would be calculated from conventional quantum mechanics (e.g. on the order of 10^{-8} sec for 'dipole'

† Leiter, D. Elementary Measurement Electrodynamics II: A Relativistic Non-linear Wave-Mechanical Theory For Many-Electron Atoms (in preparation). In this future publication, the work of the present article is generalized so as to include the effects of the Pauli Exclusion Principle, and correlation interaction effects. This is accomplished by properly antisymmetrizing the 'labeling' of the elementary measurement structure, directly in the action principle.

‡ Messiah, A. (1966). *Quantum Mechanics*, Vol. I, Chap. IV, Sections 9 and 10. John Wiley and Sons. In this reference it is shown that the Schwartz Inequality, and the constraints it puts on the solutions to the associated wave mechanics, are derivable from general principles which are independent of whether the wave-mechanical equations of motion are linear or non-linear.

transitions). This is possible in this theory because, contrary to the basic structure of conventional quantum mechanics, the measurement process is being described by *non-linear* c-number differential equations in *space-time*, rather than by linear differential equations in configuration space. It is the linear configuration space characteristics of quantum mechanical equations which prevents quantum jumps from being describable, except in statistical terms.

Radiation in this theory is defined in a similar fashion as that of action-at-a-distance electrodynamics.† When an LCT occurs, for a given hydrogen atom, it emits a sequence of 'classical' pulses in its corresponding electromagnetic field. These pulses travel outward, at the speed of light, and are eventually absorbed by other hydrogen atoms in the model universe. However, in this theory, both the 'emitter' and the 'detector' are required to describe the radiation process (which we call 'measurement radiation').‡ Hence, even though the electromagnetic pulse is a classical one, the response of the detector to this classical pulse is to make induced LCT to higher (or lower) LCS, depending on the initial state of the detector atom. Since this will change the energy of the detector atom by discrete amounts (associated with the fact that the LCS energies are discreet), then the 'measurement radiation', represented by the induced response of the detector atom, is 'quantized' in the sense that the induced response of the detector is quantized. Since measurement radiation always requires the detector for its definition (as well as for its actual physical observation in the laboratory), then measurement radiation will contain the apparent effects of 'photons' without the requirement of 'second quantization' of the associated electromagnetic field.

An interesting connection exists between this essentially deterministic theory of the elementary measurement of charged matter fields in space-time, and the statistical predictions of conventional quantum mechanics in configuration space. First of all, as was noted previously, the LCS are in direct correspondence with the conventional quantum mechanics of non overlapping Dirac hydrogen atoms. In addition, if we consider a model universe made up of a large number of hydrogen atoms, then in order to make a deterministic prediction from the theory, we must have the required initial data needed to run the large number of associated non-linear differential equations of the system in space-time. However, we will show that the measurement equations require the initial values of the matter wavefunctions *and* their derivatives. The conventional quantum mechanics of the problem would only require the initial values of the quantum mechanical wavefunction, in configuration space, in order to run the associated linear first-order quantum mechanical equation. We will show that 'elementary measurement electrodynamics', applied to wave mechanics, represents a kind of 'hidden variable' theory with respect to quantum mechanics, in that

† See Wheeler and Feynman (1949), and Leiter (1969).

‡ Refer to Leiter (1969), section on Elementary Measurement Radiation.

the greater amount of initial data (than required by conventional quantum mechanics) for the aggregate of atoms, when treated in a classical statistical sense, can yield the associated statistical predictions of quantum mechanics. Interference effects can still occur, within this model, because of the wave-mechanical structure over which the classical statistics is being taken.† In this fashion a 'probability per unit time' of an 'induced' transition and of a 'spontaneous' transition can be defined for a typical hydrogen atom in the aggregate. These quantities are then found to be identical to the 'Einstein A and B coefficients', usually derived within the framework of quantum electrodymamics. However, in this model, no 'second quantization' of the associated electromagnetic fields is ever used. The implication to be drawn from this result is that, in theories of this type, the apparent effects of 'photons' can be generated without ever introducing 'photons' as elementary particles (e.g. quanta of the associated second-quantized electromagnetic field).

Having calculated the 'Einstein A and B coefficients' of the theory, we will proceed to assume that the aggregate of hydrogen atoms is in statistical equilibrium, with respect to the mutual 'measurement radiation' energy being swapped back and forth in the model universe. Then the usual derivation of the Planck blackbody formula, utilizing the Einstein A and B coefficients in equilibrium, follows automatically for the 'measurement' radiation energy density per unit frequency, detected by a typical hydrogen atom in the model universe. This result is very interesting, in that this type of atomic theory (which falls into the category of semi-classical radiation theories) is able to account for both spontaneous and induced transitions of an aggregate of atoms, and yield from this the well-known Planck blackbody formula, *without* recourse to a underlying 'second quantization' of the associated electromagnetic fields in the theory.

In the 2nd and 3rd section of this paper we will discuss possible generalizations of this theory to many-electron atoms (where the Pauli Exclusion Principle must be accounted for)‡ and to the structure of positronium and the electron–positron interaction in general, and the role it plays (in conjunction with the Pauli effect) in avoiding the 'negative energy' collapse of relativistic atoms described by Dirac wave mechanics.

2. *Structure of The Non-linear Elementary Measurement Electrodynamic Field Theory for Non-overlapping Hydrogen Atoms*

In our model wave-mechanical theory of Elementary Measurement Electrodynamics, the matter fields will be denoted by the complex spinor fields $\psi^{(2K)}(\mathbf{x}, t)$, where $K = 1, 2, \ldots, N$, will denote the particle field label

† This is explained in detail in Section 6 of this article.

‡ If the Pauli Exclusion Principle is not properly accounted for, in any semiclassical Dirac theory for many-electron structure, then nature of this type of theory involves a tendency to collapse into a minus infinity of energy, because of the presence of negative energy solutions.

associated with the matter degrees of freedom in the associated hydrogen atom interactions. The proton degrees of freedom will be assumed to have an infinite mass associated with them, and will hence be effectively decoupled from the dynamics of the interactions.† The associated electronic current densities, in space-time, are given by

$$J_\mu^{(2K)}(\mathbf{x}, t) = -e\bar{\psi}^{(2K)}(\mathbf{x}, t)\gamma_\mu\psi^{(2K)}(\mathbf{x}, t) \qquad (K = 1, 2, \ldots, N) \quad (2.1)$$

and the related electromagnetic field degrees of freedom, related to each electronic matter field and each infinitely massive proton field, are given by $A_\mu^{(K)}(\mathbf{x}, t)$; $F_{\mu\nu}^{(K)} = A_{\mu,\nu}^{(K)} - A_{\nu,\mu}^{(K)}$ where $K = 1, 2, \ldots, N$ (there are N hydrogen atoms assumed to exist in the model universe). The 'protonic' current densities in space-time are

$$J_\mu^{(2K-1)}(\mathbf{x}, t) = (e\delta^3(x^{(2K-1)}), 0) \qquad (K = 1, 2, \ldots, N) \quad (2.2)$$

where the assumption of non-overlapping atoms‡ requires that

$$|\mathbf{x}^{(2K-1)} - \mathbf{x}^{(2J-1)}| \geqslant (\text{Bohr radius}) \qquad K \neq J = 1, 2, \ldots, 2N \quad (2.3)$$

Applying the paradigm of 'Elementary Measurement', in analogy to the method discussed in Leiter (1969), the charged matter fields $\psi^{(K)}$, and their associated current density fields $J_\mu^{(K)}$ $(K = 1, 2, \ldots, 2N)$ and electromagnetic fields $A_\mu^{(K)}$ $(K = 1, 2, \ldots, 2N)$, are not elementary entities in themselves. Instead, they are considered to be interdependent degrees of freedom in 'elementary mutual measurement fields' $J_\mu^{(K)}A^{\mu(J)}$ $(K \neq J = 1, 2, \ldots, 2N)$, which are considered to be physically more fundamental than either $J_\mu^{(K)}$ or $A_\mu^{(J)}$. Hence, on the basis of the paradigm which underlies this model theory of atomic structure, 'self-measurement' fields $J_\mu^{(K)}A^{\mu(K)}$ $(K = 1, 2, \ldots, 2N)$, are excluded, *a priori*, as being operationally unphysical.

Then in terms of these field degrees of freedom, the action principle, built out of the possible 'elementary measurement' interactions available, is

$$I = \int dx^4 \left[\sum_{K=1}^{N} \mathscr{L}_{\text{Dirac}}(\psi^{(2K)}) + \sum_{K=1}^{2N} \sum_{J \neq K=1}^{2N} \left\{ \frac{F_{\mu\nu}^{(K)} F^{\mu\nu(J)}}{4} + J_\mu^{(K)} A^{\mu(J)} \right\} \right] \quad (2.4)$$

$$= \int dx^4(\mathscr{L})$$

† This is because the strength of the *non-linear* electromagnetic interaction (i.e. that part in addition to the coulomb interaction) will be proportional to the ratio of the masses of the electron to the proton. As the proton mass is allowed to become infinite, this non-linearity vanishes, and the proton behaves as if it is 'external' to the system. Of course, this situation is modified if the proton is allowed to have a dynamic 'structure' (see footnote on p. 218).

‡ This approximation is required in our model, since we neglect the effects of the Pauli Principle. Inclusion of the Pauli Effect, as discussed in footnote on p. 208 and footnote on p. 217. would allow us to relax this assumption, if we wished, but would not alter the results of our present theory. This is because the physical effects of the Pauli Principle are negligible for non-overlapping atoms anyway, and the generalized theory would yield the same results as that of Section 2, since only non-overlapping atoms are dealt with there.

where $\mathscr{L}_{\text{Dirac}}(\psi^{(K)})$ is the Dirac Lagrangian density†

$$\mathscr{L}_{\text{Dirac}}(\psi^{(2K)}) = \tfrac{1}{2}[\bar{\psi}^{(2K)}(-i\,\partial\mu\gamma^{\mu} + m)\psi^{(2K)} + \text{h.c.}] \tag{2.5}$$

and units with $\hbar = c = 1$; $\alpha = e^2/4\pi$; m, e = physical mass, charge are used. Since the proton mass is taken to be infinite, it is effectively decoupled from the dynamics of the interaction, in this approximation, hence only the electron matter fields $\psi^{(2K)}(\mathbf{x}, t)$; $K = 1, 2, \ldots, N$ and the electronic-protonic electromagnetic fields $A_{\mu}^{(K)}$; $K = 1, 2, \ldots, 2N$ are dynamical variables in this model. The currents in (2.4) are, of course, given by (2.1) and (2.2). We emphasize, that in the action principle (2.4), all 'self-measurement' fields, $J_{\mu}^{(K)} A^{\mu(K)}$ are excluded, *a priori*, from the system. This is as required by the basic paradigm of the theory which states that it is only the 'mutual measurement interactions' which are the basic building of the physical system.

Setting $\delta I = 0$ with respect to the dynamical variables, yields the equations of motion of the elementary measurement fields, representing the interaction of N non-overlapping hydrogen atoms in space-time, are

$$\left[-i\,\partial\mu\gamma^{\mu} + m - e \sum_{J \neq 2K=1}^{2N} (A_{\mu}^{(J)} \gamma^{\mu})\right]\psi^{(2K)} = 0 \qquad (K = 1, 2, \ldots, N) \tag{2.6}$$

and the corresponding Maxwell equations

$$\sum_{J \neq K=1}^{2N} F_{\mu\nu}^{(J),\nu} = \sum_{J \neq K=1}^{2N} J_{\mu}^{(J)} \qquad (K = 1, 2, \ldots, 2N) \tag{2.7}$$

$$F_{\mu\nu}^{(K),\nu} \equiv 0 \tag{2.8}$$

Equations (2.6) and (2.7) represent together a set of N coupled, non-linear integro-differential equations in four-dimensional space-time. The integro-differential character will become more explicit when we solve the Maxwell equations for $A_{\mu}^{(J)}$ in terms of $J_{\mu}^{(J)}$, and substitute this into (2.6). However, before carrying this procedure out, we note that in keeping with the basic paradigm, the Maxwell equations of this model are to be considered as four-vector connections, which give a prescription for converting $A_{\mu}^{(J)}$ into their associated $J_{\mu}^{(J)}$. This will require that all homogeneous solutions to (2.7), *unconnected from currents*, must be omitted as being unphysical, since they do not represent 'mutual' elementary measurement interactions when coupled to other fields in the system‡. To do this

† We essentially use the Dirac Bispinor notation of Schweber, S. (1961). *Introduction To Relativistic Quantum Field Theory*, (1961). Chap. 3. Row-Peterson.

‡ This is because homogeneous solutions to the associated Maxwell equations, uncoupled from currents, while generally present (for mathematical reasons), may be excluded (for physical reasons). Since elementary measurement interactions involve (in this theory) the interaction of electric currents with electric currents, then the interactions of electric currents with homogeneous potentials, unconnected to currents, must be excluded in this theory in order to be consistent with the basic paradigm. We refer the reader to the article by Leiter (1969) for the classical implications of this assumption.

we note that if we choose the Maxwell vector potentials to obey the Lorentz condition

$$A_{\mu}^{(J),\mu} = 0 \tag{2.9}$$

then (2.7) becomes

$$\sum_{J\neq K=1}^{2N} \Box A_{\mu}^{(J)} = \sum_{J\neq K=1}^{2N} J_{\mu}^{(J)} \qquad (K = 1, 2, \ldots, 2N) \tag{2.10}$$

If there is at least one hydrogen atom in our model universe ($N \geqslant 1$), then (2.10) can be solved algebraically for the individual $\Box A_{\mu}^{(J)}$ in terms of the individual $J_{\mu}^{(J)}$, since the determinant of the coefficients of $\Box A_{\mu}^{(J)}$ is non-zero in (2.10), as

$$\Box A_{\mu}^{(J)} = J_{\mu}^{(J)} \qquad (J = 1, 2, \ldots, 2N) \tag{2.11}$$

We can solve (2.11) for the solutions which are always coupled to currents by using the Green function $D^{(JK)}(x - x')$, as

$$A_{\mu}{}^{J}(x) = \int dx'^{4} \sum_{K=1}^{2N} D^{(JK)}(x - x') J^{(K)}(x') \tag{2.12}$$

where $D^{(JK)}$ satisfies the equation

$$\Box D^{(JK)}(x - x') = \delta^{4}(x - x')\, \delta^{JK} \tag{2.13}$$

Equation (2.12) will automatically omit homogeneous solutions, *unconnected from currents*, as required by the basic paradigm.† The most general solution to (2.13) is

$$D^{(JK)}(x - x') = D_{+}(x - x')\, \delta^{JK} + D_{-}(x - x')\, \lambda^{JK} \tag{2.14}$$

where

$$D_{\pm}(x - x') = \frac{D_{\text{ret.}}(x - x') \pm D_{\text{Adv.}}(x - x')}{2} \tag{2.15}$$

and λ^{JK} is an arbitrary constant matrix, to be determined empirically. Substituting equations (2.12) through (2.15) into (2.6) yields the non-linear integro-differential equations of motion of the measurement,

$$\left[-i\partial\!\!\!/ + m - e\left\{\sum_{J\neq 2K=1}^{2N} A^{(+)(J)} + \sum_{J\neq 2K=1}^{2N} \sum_{L=1}^{2N} \lambda^{JL} A^{(-)(L)}\right\}\right] \psi^{(2K)} = 0 \tag{2.16}$$

$$(K = 1, 2, \ldots, N)$$

where $A_{\mu}^{(K)} \gamma^{\mu} \equiv A^{(K)}$; $i\partial\!\!\!/ \equiv i\partial_{\mu} \gamma^{\mu}$ and

$$A_{\mu}^{(\pm)(J)}(x) \equiv \int dx'^{4}\, D_{\pm}(x - x') J_{\mu}^{(J)}(x') \qquad (J = 1, 2, \ldots, 2N) \tag{2.17}$$

† This occurs, even though [in equations (2.12) and (2.14)] $A_{(-)}^{\mu(J)}$ obeys the homogeneous maxwell equation. This is because $A_{(-)}^{\mu(J)}$, being defined in terms of the current $J_{\mu}^{(J)}$, *is not a homogeneous solution unconnected from currents*. Hence the interactions it plays a role in, will be different than that generated between homogeneous potentials, unconnected from currents, and currents. For this reason $A_{(-)}^{(\mu)}$ is admissible under the basic paradigm of the theory.

Since $A^{(J)}_{\mu(\text{ret.})} = A^{(J)(+)}_{\mu} + A^{(J)(-)}_{\mu}$, then (2.16) can be written in the form $(K = 1, 2, \ldots, N)$

$$\left[-i\partial\!\!\!/ + m - e\left\{\sum_{J\neq 2K=1}^{2N} A^{(J)}_{(\text{ret.})} + \sum_{J\neq 2K-1}^{2N}\left(\sum_{K=1}^{2N}\lambda^{JL}A^{(-)(L)} - A^{(J)(-)}\right)\right\}\right]\psi^{(2K)} = 0 \tag{2.18}$$

Now (2.18) represents a class of electromagnetic theories (parameterized by λ^{JL}) for which the correct correspondence to the predictions of classical electrodynamics will occur if γ^{JL} is chosen so that†

$$\sum_{J\neq 2K=1}^{2N}\left(\sum_{K=1}^{2N}\lambda^{JL}A^{(L)(-)} - A^{(J)(-)}\right) = A^{(2K)(-)} \tag{2.19}$$

To see that this choice is possible, we note that (2.19) can be written in terms of the $2N \times 2N$ matrices $\Omega^{KJ} \equiv (1 - \delta^{KJ})$; λ^{KJ} and the $2N$-column $A_{(-)}$ (made up from $A^{(K)}_{(-)}$) as

$$[(\Omega\lambda - \Omega)A]^{2K\text{th component}} = A^{(2K)}_{(-)}. \tag{2.20}$$

This will be satisfied if‡

$$\Omega\lambda - \Omega = I; \qquad \lambda = \Omega^{-1}(\Omega + I) \tag{2.21}$$

is chosen, as it can be since $|\Omega| \neq 0$ implies that Ω^{-1} exists. Hence, with the choice of $\lambda^{JL} = (\Omega^{-1}(\Omega + I)^{JL}$ as implied by (2.21), the equations of motion of the elementary measurement of N non-overlapping hydrogen atoms is, from (2.18)

$$\left[-i\partial\!\!\!/ + m - e\left\{\sum_{J\neq 2K=1}^{2N}(A^{(J)}_{\text{ret.}}) + A^{(-)(2K)}\right\}\right]\psi^{(2K)} = 0 \qquad (K = 1, 2, \ldots, N) \tag{2.22}$$

In a sense, (2.22) represents a kind of wave-mechanical generalization of 'action-at-a-distance' electrodynamics, except that no 'complete absorber' condition is ever used,§ and the electromagnetic fields are not eliminated *a priori*. It is the solutions of these N non-linear, partial integro-differential equations which will describe the physical interaction of the N atoms in

† This was shown by Leiter (1969), where it was proven that this choice of λ^{KJ} implied that the theory contained a 'total coupled radiation field', directly connected to the currents in the elementary measurement interactions. Interference between this total coupled radiation field, and that of the time-symmetric, mutual interaction fields, were shown to yield the conventional retarded electromagnetic fields, and the radiation reaction field, if more than one charged particle existed in the model universe. That this radiation reaction effect was not due to 'self-interaction' (in the conventional sense) was shown by the fact that if the model universe contained only one charged particle, then the radiation reaction field, acting on this particle, was zero. In order to have the proper classical correspondence limit between our Dirac bispinor theory of elementary measurement electrodynamics, and that of the classical theory given by Leiter (1969), it is natural to make the analogous choice of λ^{KJ}.

‡ See Leiter (1969), and Section 2 of the paper quoted therein.

§ See Leiter (1969), Section 2.

space-time. However, before attempting to solve (2.22) we must explain how we extract physical information about observables from such solutions.

In general, the prescription for calculation of observables in Elementary Measurement Electrodynamics, in a given 3-space volume V, arises from the differential conservation laws obeyed by the measurement energy-momentum tensor of the theory.† Since this particular model theory is invariant to time translations [but not space translations, because of the 'external' character of the protonic currents (2.2)], then the relevant differential conservation law of the theory is‡

$$T^{\mu 0}{}_{,\mu} = 0 \tag{2.23}$$

where the canonical measurement energy-momentum tensor of the theory is given by

$$T^{\mu\nu} = \left[g^{\mu\nu}\mathscr{L} - \sum_{K=1}^{N} \left\{ \frac{\partial \mathscr{L}}{\partial(\partial_\mu \psi^{(2K)})} \partial^\nu \psi^{(2K)} + \text{h.c.} \right\} - \sum_{K=1}^{2N} \left\{ \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_\beta^{(K)})} \partial^\nu A_\beta^{(K)} \right\} \right] \tag{2.24}$$

The integral conservation law implied by (2.23) integrated over a given 3-space volume V, enclosed by a two-dimensional closed surface S, is

$$\partial_t E = -\int_S dS^i T^{i0} \tag{2.25}$$

where $\epsilon = \int_V dx^3 T^{\infty}$ is the total measurement energy, of the system of hydrogen atoms, contained in the volume V. In particular, (2.23) through (2.25) imply that the total measurement energy contained in V is

$$E = \int_V dx^3 \left[\sum_{K=1}^{N} (\psi^{(2K)+}(\boldsymbol{\alpha}\cdot\hat{\mathbf{p}} + \beta m)\psi^{(2K)}) + \sum_{\substack{K=1 \\ J\neq K=1}}^{2N} \left(\frac{F^{\mu\nu(K)} F^{(J)}_{\mu\nu}}{4} + J_\mu^{(K)} A^{\mu(J)} \right) - \sum_{K=1}^{2N} \sum_{J\neq K=1}^{2N} (F^{(J)}_{\beta 0} A^{(K)}_{\beta 0}/2) \right] \tag{2.26}$$

It is to be noted that no self-energy appears in (2.26), because, on the basis of the paradigm of 'Elementary Measurement', only mutual measurement interaction fields go into the action of the elementary measurement theory. It is in terms of the total measurement energy of the system that we will discuss the solutions to (2.22) in the next sections.

† See Leiter (1969), Section 3.

‡ This means that the theory will become compatible with overall conservation of momentum (and angular-momentum) only if the proton is given a dynamical structure. This shortcoming is shared with any conventional quantum theory of atoms which treats the proton as a point charge with infinite mass (essentially external to the dynamics of the theory). See footnote on p. 218.

3. *Limit Cycle Solutions of the Non-linear Field Equations and the Quantum Levels of Atomic Hydrogen*

We shall begin our discussion of the solutions to (2.22) with the especially simple case of a model universe with a single hydrogen atom in it (the case where $N = 1$). Then (2.22) becomes

$$[-i\partial\!\!\!/ + m - eA\!\!\!/^{(1)}_{(\text{ret.})} - eA\!\!\!/^{(2)}_{(-)}]\psi^{(2)}(\mathbf{x}, t) = 0 \tag{3.1}$$

where $K = 2$ denotes the electron and $K = 1$ denotes the infinitely massive proton. The associated Maxwell fields are given by (2.17) as

$$\begin{aligned} A^{(1)\mu}_{\text{ret.}}(x) &= \int dx'^4\, D_{\text{ret.}}(x - x')J^{(1)}(x')^\mu \\ A^{(2)\mu}_{(-)}(x) &= \int dx'^4\, D_{(-)}(x - x')J^{(2)}(x')^\mu \end{aligned} \tag{3.2}$$

where the related currents are from (2.1) and (2.2)

$$\begin{aligned} J^{(1)}_\mu(x) &= (e\,\delta^3(x - x^{(1)}); 0) \\ J^{(2)}_\mu(x) &= -e\bar{\psi}^{(2)}\gamma_\mu\psi^{(2)} \end{aligned} \tag{3.3}$$

Insertion of (3.3) into (3.1) yields the non-linear integro-differential equation (with retardation and advancement) as

$$\left[-i\partial\!\!\!/ + m\frac{-\gamma^0\alpha}{|\mathbf{x} - \mathbf{x}^{(1)}|} + e^2\int dx^{4\prime}\,\gamma^\mu\, D_{(-)}(x - x')\bar{\psi}^{(2)}(x')\gamma_\mu\psi^{(2)}(x')\gamma^\mu\right] \times \psi^{(2)}(x) = 0 \tag{3.4}$$

In (3.4), $\alpha = e^2/4\pi$ is the fine structure constant, and m is the empirical mass of the electron (there is no mass renormalization since there is no self-interaction).†

A possible solution to (3.4) is an LCS of the form

$$\psi^{(2)}_n(\mathbf{x}, t) = \chi^{(2)}_n(\gamma)\exp(-iE^{(2)}_n t) \tag{3.5}$$

where $\chi^{(2)}_\eta(\mathbf{x})$ obeys

$$\left[\boldsymbol{\alpha}\cdot\hat{\boldsymbol{\rho}} + \beta m - \frac{e^2}{4\pi|\mathbf{x} - \mathbf{x}^{(1)}|}\right]\chi^{(2)}_n(\mathbf{x}) = E^{(2)}_n\chi^{(2)}_n(\mathbf{x}) \tag{3.6}$$

(η is a discreet eigenlabel)

This is possible since (3.5) implies that $A\!\!\!/^{(2)}_{(-)}$ vanishes in that state. From (3.6) we see that the LCS are identical to the discreet atomic eigen-states of the Dirac equation for hydrogen, and hence the energy levels, associated with (3.6), are the Dirac eigenvalues. This means that, even though the LCS are solutions to an inherently non-linear differential equation, they nevertheless form a discreet complete set of functions which span the

† This occurs because of the absence of direct self-interaction in (2.22), and (3.1). A similar effect occurs in the classical limit (see Leiter, 1969).

discreet function space of the theory described by (3.1). This property will be used in the next section, where we discuss LCT solutions. For our present purposes we note that the LCS can be assumed to obey the orthogonality condition

$$\langle \chi_n^{(2)} / \chi_{n'}^{(2)} \rangle = \delta_{nn'} \tag{3.7}$$

since the LCS equation (3.6) is essentially a linear one. In this case, the total measurement energy of the hydrogen atom, given by (2.26), is

$$\begin{aligned} E_n &= \int dx^3 [\chi_n{}^{+(2)} (\boldsymbol{\alpha} \cdot \hat{\boldsymbol{\rho}} + \beta m - e^2/4\pi |x - x^{(1)}|) \chi_n^{(2)}] \\ &= E_n^{(2)} \end{aligned} \tag{3.8}$$

in agreement with the LCS energy associated with the measurement equations of motion (in LCS), given by (3.5) and (3.6), We note that (2.25) also implies that in LCS, $\partial_t E_\eta = 0$, while (3.6) implies that the LCS are derivable from the time-independent variational principle

$$\delta[E_n - \lambda \langle \chi_n^{(2)} / \chi_n^{(2)} \rangle] = 0 \tag{3.9}$$

subject to the constraint (3.7).

Since this model deals with non-overlapping hydrogen atoms only, then the effects of the Pauli Principle (required for electron–electron interactions) and the effects of 'electron correlations' are neglected. However, they can be included in a more general version of this model theory by extending it to include the required elementary measurement interactions needed to account for these physical effects.† In addition, if the full electron–positron elementary measurement interactions are taken into account, in such a generalized version of this theory, the remaining accidental degeneracies in the Dirac energy levels in (3.6) are broken by the coupling to the 'annihilation state' of positronium present in such a model.‡ Finally, if the assumption of an infinitely massive proton is relaxed, a 'reduced mass' effect on the associated LCS energies will occur, but may require the assumption of 'proton structure' (due to 'strong' elementary measurement

† In the author's generalization of the present theory, the Pauli Principle, *and* the effects of 'correlation interaction', are introduced by properly anti-symmetrizing the elementary measurement interactions directly in the Lagrangian formalism *a priori* This approach automatically carries this anti-symmetry into the associated energy-momentum tensor, and hence implies that matter-wave electron functions with identical LCS, are not possible extrema of the total energy.

‡ In the generalized version of this theory (referred to in footnote on p. 208) the 'vacuum' is postulated to be a sea of 'electron-positron' elementary measurement interactions in their respective 'annihilation states'. This vacuum state has zero total energy and momentum, but is capable of interacting with atoms via a 'vacuum polarization' effect. This mechanism will tend to break the degeneracy of the hydrogenic levels described by (3.1) in LCS, and will also prevent a 'negative energy collapse', of the ground state of hydrogen, into the negative continuum of energy (contained in the relativistic Dirac wave-mechanical structure) by virtue of the antisymmetrized elementary measurement interactions (which produce the Pauli Principle) in the theory. Although, at the time of writing, a calculation of this 'vacuum' induced energy level shift in hydrogen has not yet been carried out, we speculate that this mechanism may be able to account for the Lamb Shift in this elementary measurement theoretical context.

interactions) in order to agree with the physically observed effects of the reduced mass on the hydrogenic energy levels.†

Neglecting the above-mentioned effects, in our present model theory, we will study the physical properties of LCS [and a second type of solution to (2.22), LCT solutions] and show that even though the structure of this theory is different from conventional quantum mechanics, there is a relationship between the physical predictions of this non-linear wave theory in space-time, and conventional quantum theory in configuration space.

As a final remark in this section, we note that if we had tried to formulate this theory within the framework of conventional Maxwell–Lorentz electrodymamics, then the resulting non-linear equations corresponding to (2.22) and (3.1) would have included the effects of the direct self-interaction field. $A^{2K}_{(+)}$.‡ This self-interaction term would *prohibit* LCS type of solutions from existing, since the balance between the electromagnetic forces and the 'Schwartz Inequality' forces (inherent in the wave-mechanical structure) would be destroyed. This balance is required in order for the LCS eigenfunction solutions to exist. It is interesting to speculate that the reason why the relationship between LCS and quantum levels was not realized earlier might be traced to the fact that Maxwell–Lorentz electrodymamics (which automatically includes direct self-interactions) when combined with wave mechanics, yields a non-linear, wave-mechanical theory which prevents LCS from existing.

4. *Limit Cycle Transition Solutions (LCT), 'Quantum Jumps', and the Lifetime of Excited (LCS)*

In the non-linear, wave-mechanical, elementary measurement theory under consideration, in addition to the LCS solutions (3.5), of equation (3.1), there exist LCT types of solutions. We obtain these solutions by noting that the fact that the LCS form a discreet, complete set, over the discreet part of the non-linear function space of the theory, allows us to express solutions to (3.1), which span that region of the function space, as

† If the proton is given a dynamical role in the theory, then it must participate in more than just purely electromagnetic interactions. This is because the Schwartz Inequality implies that $\alpha \ll 1$ is too small to localize the proton into a region $\sim 10^{-13}$ cm, in an atomic LCS. Hence, an obvious need for an additional 'strong interaction' (i.e. with an effective coupling constant g greater than unity) makes its appearance in Elementary Measurement theory. If such a 'strong interaction' is postulated then the proton's finite mass will produce a reduced mass correction to the hydrogenic energy levels, obtained from the total energy of the system. The details of this calculation will be discussed in a later publication.

‡ This can be seen by noting that the conventional Lorentz–Dirac equation for classical electrodynamics of point charges in Maxwell–Lorentz Theory (see Jackson, J. D. (1963). *Classical Electrodynamics*, Chap. 17. John Wiley and Sons) with the assumption of retardation, implies that in addition to the radiation reaction force, proportional to $A^{\mu}_{(-)}$, there is the time-symmetric self-interaction $A^{\mu}_{(+)}$ (which is then absorbed into the bare mass by a mass-renormalization technique). This would also occur in any theory which attempted to make a union between Maxwell–Lorentz electrodynamics and wave mechanics, because of the presence of self-interactions in such theory.

a superposition of LCS with time-dependent coefficients, in the following manner†

$$\psi^{(2)}(\mathbf{x}, t) = \sum_n c_n^{(2)}(t)\, \chi_n^{(2)}(\mathbf{x}) \tag{4.1}$$

where

$$\sum_n |c_n^{(2)}(t)|^2 = 1$$

is preserved in time. Substitution of (4.1) into (3.1) yields the non-linear equations of motion for the coefficients as

$$i\dot{c}_n^{(2)}(t) = \sum_m \Gamma_{nm}^{(2)}(t)\, c_m^{(2)}(t) \tag{4.2}$$

where

$$\Gamma_{nm}^{(2)}(t) = E_m^{(2)}\, \delta_{nm} + \langle \chi_n^{(2)} | - e\mathcal{A}_{(-)}^{(2)} | \chi_m^{(2)} \rangle \tag{4.3}$$

If we define the density matrix of the electron matter field as

$$\rho_{nm}^{(2)}(t) \equiv c_n^{(2)}(t)\, c_m^{(2)*}(t) \tag{4.4}$$

then the density matrix equations of motion of the measurement are from (4.2)

$$i\dot{\rho}_{nm}^{(2)} = \sum_l (\Gamma_{nl}^{(2)}\, \rho_{lm}^{(2)} - \rho_{nl}^{(2)}\, \Gamma_{lm}) \tag{4.5}$$

It should be noted that (4.2) and (4.5) yield the LCS results of (3.5), when $c_{n'}^{(2)} \to \exp(-iE_{n'}^{(2)}t)x\delta_{n'n}$ and $\rho_{n'm'}^{(2)} \to \delta_{n'n}\delta_{m'n}$. Hence the LCT solutions to (4.5) give a causal, space-time description of the time evolution of the hydrogen atom, as it 'jumps' from one LCS to another. Since the LCS play the role of quantum levels in this type of measurement theory, then the LCT give a causal description of 'quantum jumps' in this model formalism. This is possible since, in contradistinction to the conventional quantum mechanics of coulombic hydrogen, the elementary measurement wave-mechanical hydrogen atom is being described by a *non-linear* integro-differential equation in space-time (in conventional quantum theory, the atom would be described by a *linear* differential equation in the associated configuration space). For this reason, non-linear elementary measurement theories of this type are fundamentally quite different from orthodox

† Of course, in general, we should also include in our expansion the corresponding positive and negative energy coulomb continuum states implied by (3.1). By assuming that the atom is described by (4.1), we imply that no ionization is occurring (no positive energy coulombic continuum state contribution), and that the negative energy continuum is somehow uncoupled from the dynamics. In the present theory, this latter assumption is not true and the ground LCS is unstable to a negative energy collapse into minus infinite energy. However, in the generalized version of this theory (see footnote on p. 208 and footnote on p. 217) the presence of the 'vacuum' sea of positronium atoms in their annihilation states, and the Pauli interaction generated by the anti-symmetrized elementary measurement interactions, imply that the hydrogen atom is effectively decoupled from its negative energy continuum. This is because if the electron entered a negative energy state it would have the same properties as the positrons in the 'vacuum sea', which is dynamically prohibited by the properly anti-symmetrized elementary measurement interaction structure. This generalized version of the theory would then yield the same results as the present theory, with the assumption (4.1), and justifies our use of (4.1) in Section 4.

quantum mechanics, and instead represent a kind of hybrid union of classical electrodynamics and wave mechanics (e.g. a semi-classical type of structure) for non-linear charged Dirac matter waves in space-time.

Because of the unorthodox structure of this elementary measurement theory of coulombic hydrogen, the LCT solutions to (3.1) describe the process of transition between LCS in a causal space-time picture. Hence it is of great interest to determine the time of evolution of an LCT into an LCS. We can obtain this information from (4.5) by inserting (4.3) and (2.17) and expanding the potentials in powers of $|\mathbf{x}-\mathbf{x}'|/c$ (allowable since $\alpha = 1/137$, is so very much less than unity). Then to lowest order in $|\mathbf{x}-\mathbf{x}'|/c$ equation (4.3) becomes

$$\Gamma^{(2)}_{nm} = \frac{\alpha}{c^3}\left[\frac{\mathbf{x}^{(2)}_{nm}}{(3)}\cdot\sum_{p_q}\mathbf{x}^{(2)}_{pq}\,\ddot{\rho}^{(2)}_{qp} + i\Delta^{(2)}_{nm}\,\mathbf{x}_{nm}\cdot\sum_{p_q}\mathbf{x}_{pq}\,\ddot{\rho}^{(2)}_{qp} + \delta_{NM}\sum_{p_q}\frac{(\mathbf{x}\cdot\mathbf{x})_{pq}}{12}\,\ddot{\rho}^{(2)}_{qp}\right] + E^{(2)}_{m}\,\delta_{nm} \tag{4.6}$$

where

$$\Delta^{(2)}_{nm} = E^{(2)}_{n} - E^{(2)}_{m}; \qquad \mathbf{x}^{(2)}_{nm} = \langle\chi^{(2)}_{n}|\mathbf{x}|\chi^{(2)}_{n}\rangle$$

Inserting (4.6) into (4.5) yields

$$i\dot{\rho}^{(2)}_{nm} = \Delta^{(2)}_{nm}\,\rho^{(2)}_{nm} + \sum_{l}\left[\Gamma^{(2)\prime}_{nl}\,\rho^{(2)}_{lm} - \rho^{(2)}_{nl}\,\Gamma^{(2)\prime}_{lm}\right] \tag{4.7}$$

$$\Gamma^{(2)\prime}_{nl} = \frac{\alpha}{c^3}\left[\frac{\mathbf{x}^{(2)}_{nm}}{(3)}\cdot\sum_{p_q}\mathbf{x}^{(2)}_{pq}\,\ddot{\rho}^{(2)}_{qp} + i\Delta^{(2)}_{nM}\,\mathbf{x}^{(2)}_{nm}\cdot\sum_{p_q}\mathbf{x}^{(2)}_{pq}\,\ddot{\rho}^{(2)}_{qp}\right]$$

Because of the smallness of $\alpha = 1/137$, the second and third derivative terms in (4.7) are much smaller than the main frequency term, involving no derivative. Hence $\dot{\rho}^{(2)}_{nm} \simeq -i\Delta^{(2)}_{nm}\rho^{(2)}_{nm}$ is a good approximation which, when differentiated twice, implies that $\ddot{\rho}^{(2)}_{nm} \simeq -[\Delta^{(2)}_{nm}]^2\rho^{(2)}_{nm}$ and $\dddot{\rho}^{(2)}_{nm} \simeq i(\Delta^{(2)}_{nm})^3\rho^{(2)}_{nm}$. Substitution of this approximation of the derivative terms into (4.7) yields for $\Gamma^{(2)\prime}_{nl}$

$$\Gamma^{(2)\prime}_{nm} \simeq \frac{\alpha}{c^3}\left[\frac{\mathbf{x}^{(2)}_{nm}}{(3)}\cdot\sum_{p_q} i[\Delta^{(2)}_{qp}]^3\,\mathbf{x}_{pq}\,\rho^{(2)}_{qp} - i\Delta^{(2)}_{nm}\,\mathbf{x}^{(2)}_{nm}\cdot\sum_{p_q}(\Delta^{(2)}_{qp})^2\,\mathbf{x}^{(2)}_{pq}\,\rho^{(2)}_{qp}\right] \tag{4.8}$$

Again because of the smallness of α, the $\rho^{(2)}_{nm}(t)$ undergo oscillations, during LCT, which have a very slowly varying envelope. For this reason, if T represents a time interval which contains a large number of characteristic periods of oscillation, then

$$\overline{\rho^{(2)}_{nm}\,\rho^{(2)}_{pq}} = \frac{1}{T}\int_0^T \rho^{(2)}_{nm}\,\rho^{(2)}_{pq}\,dt \simeq (\delta nq\,\delta mp + \delta nm\,\delta pq) \tag{4.9}$$

Using (4.9), if we calculate the time average of (4.7) over this interval T, then we have for the case where $n = m$,

$$\overline{\dot{\rho}^{(2)}_{nn}} \simeq \frac{4\alpha}{3c^3}\sum_{l}(\Delta^{(2)}_{ln})^3|\mathbf{x}^{(2)}_{ln}|^2 \tag{4.10}$$

Since $\rho_{\eta\eta}^{(2)}$ represents the weighting of the ηth LCS in the ensuing LCT, then the characteristic time it takes for the atom to evolve from the rth LCS into a state where $\rho_{rr}^{(2)} = 0$, is given by the reciprocal of (4.10) with n set equal to r,† as

$$\tau_r^{(2)} \simeq |1/\dot{\rho}_{rr}^{(2)}| \simeq \left| \left[\sum_{l=1}^{r} \frac{4\alpha}{3c^3} (\Delta_{lr}^{(2)})^3 |\mathbf{x}_{lr}^{(2)}|^2 \right] \right|^{-1} \tag{4.11}$$

The characteristic time of energy transfer during the LCT from the rth LCS, given by (4.11), is in exact agreement with the lifetime of the rth excited quantum level (of a hydrogen atom) which is obtained from orthodox quantum mechanics, utilizing a 'second quantization' of the electromagnetic field.‡ The implication here is that the time required for a 'quantum jump' to occur, in elementary measurement theory, is precisely the observed 'lifetime' of the associated quantum states involved. The reason why these 'spontaneous transitions' can occur in this 'semi-classical' type of elementary measurement theory, is because of its inherent non-linear structure. The non-linearity implies that the excited LCS represent a form of unstable equilibrium. Hence, if the atom is only approximately in the rth LCS at $t = 0$, because of other disturbances in the universe,§ the atom in question will have a strong tendency to evolve (via LCT) to LCS of lower energy, all this occurring without ever 'second quantizing' the associated electromagnetic field.

However, it is experimentally well known that electromagnetic radiation emitted from atoms appears quantized into 'photons' of electromagnetic energy. If elementary measurement theory does not quantize the electromagnetic field, then how does it account for the apparent corpuscular character of atomic radiation? We shall answer this question in the next section, where we discuss 'measurement radiation'.

5. *'Measurement Radiation', 'Photons', and Induced Limit Cycle Transitions*

In the previous section, we showed that an isolated hydrogen atom has a strong tendency to make limit cycle transitions to lower energy LCS, if it is in an excited LCS, because of the inherently nonlinear structure of the equations of motion of the hydrogen atom in this model. However, when this LCT occurs a 'pulse' is generated in the mutual retarded potential $A_{\mu(\text{ret.})}^{(2K)}$ (assuming that the Kth atom is in LCT). This pulse travels outward

† This is because we essentially assume that (4.1) is valid with the boundary condition that the atom is in the rth LCS at $t = 0$. Then the reciprocal of the time average of $\dot{\rho}_{rr}^{(2)}$ will give the characteristic relaxation time of the rth state to decay into $\rho_{rr}^{(2)} = 0$.

‡ A straightforward discussion of 'second quantization' is given in Gottfreid, K. (1966). *Quantum Mechanics*, Chap. VIII. W. A. Benjamin.

§ These 'other disturbances' would be properly accounted for by the presence of the 'vacuum' discussed in footnotes on pp. 217, and 219. Another possibility would be from the electromagnetic disturbances due to other atoms in the model universe.

with the speed of light, until it is 'detected' by another atom $\psi^{(2J)}$; $(J \neq K)$. Since the act of detection of the pulse in $A^{(2K)}_{\mu(\text{ret.})}$ *requires* a 'detector' atom to be present, the LCT pulse in $A_{\mu(\text{ret.})}^{(2K)}$ *by itself* does not represent the radiation process. This is because, in Elementary Measurement Electrodynamics, the measurement interaction between emitter and detector *cannot* be neglected. This process is similar to the definition of radiation in 'action-at-a-distance' types of electrodynamics, but in this theory no 'complete absorber' condition is invoked, and the electromagnetic fields are dynamical variables not eliminated *a priori*.† We shall refer to this process of measurement of radiation as 'measurement radiation', to distinguish it from that usually referred to in conventional Maxwell–Lorentz types of electrodynamics (where the 'detector' is implicitly assumed to vanish in the limit).‡

Within the framework of our theory, we see that even though the pulse is a classical pulse, the induced response of the 'detector' atom, to this classical pulse, is to make LCT to higher or lower LCS (depending on the initial state of the detector and the phase of the pulse when it arrives at the detector). *Since 'measurement radiation' is defined by the induced response of the 'detector' to the 'emitter' field*, and since the energy of the 'detector' is changed by discreet amounts when an LCT occurs, then the 'measurement radiation' is apparently 'quantized', in the sense that the induced response of the 'detector' is quantized. Since measurement radiation always requires the detector for its definition, then measurement radiation contains the apparent effects of 'photons'. However, no 'second quantization' has been invoked and these 'photons' are not elementary particles. Instead, they are secondary dynamical effects associated with the structure of the elementary measurement. The conclusion to be drawn here is not that 'second quantization' is 'wrong', but rather that the apparent effects of 'photons' can be produced, in wave-mechanical Elementary Measurement Electrodynamic theories, without requiring the 'second quantization' formalism.

To see this process in more detail, we shall examine the process of measurement radiation in terms of two atoms $\psi^{(2)}(\mathbf{x},t)$; $\psi^{(4)}(\mathbf{x},t)$ localized at $\mathbf{x}_{(1)}$; $\mathbf{x}_{(3)}$ where $|\mathbf{x}_3| \gg |\mathbf{x}_1|$. Then if $\psi^{(2)}(\mathbf{x},t)$ is undergoing an LCT, the process is described by (2.22) with $N=4$. Now at a time $|\mathbf{x}_3|/c$ after the LCT of $\psi^{(2)}(\mathbf{x},t)$ begins (we will assume that the origin or our coordinates is at $\mathbf{x}_1 \equiv 0$), $\psi^{(4)}(\mathbf{x},t)$ will begin to sense the wave-zone component of $A^{(2)}_{\mu(\text{ret.})}$ in its equation of motion. From (2.17), and the fact that $A_{\mu(\text{ret.})} = A_{\mu(+)} + A_{\mu(-)}$, this wave-zone field is given by

$$A^{(2)}_{\mu(\text{ret.})}(\mathbf{x},t) \simeq \frac{-e}{4\pi|\mathbf{x}|}\int \mathrm{d}x'^3 \bar{\psi}^{(2)}(\mathbf{x}',t_r)\gamma_\mu \psi^{(2)}(\mathbf{x}',t_r) \qquad (5.1)$$

$$t_r = t - |\mathbf{x}|/c$$

† See Leiter (1969).

‡ In Maxwell–Lorentz electrodynamics, the electromagnetic field is defined in terms of the force on a test charge, in the limit as the test charge-to-mass ratio vanishes. This definition is clearly non-operational in the microscopic domain.

the equation of motion of the 'detector' $\psi^{(4)}$, for $t > |\mathbf{x}_3|/c$, is given by

$$\left[-i\partial\!\!\!/ + m - e \sum_{J\neq 4=1}^{4} A^{(J)}_{(\text{ret.})} - eA^{(4)}_{(-)}\right]\psi^{4}(\mathbf{x}, t) = 0 \tag{5.2}$$

Now, there will be a sequence of pulses in $A^{(2)}_{\mu(\text{ret.})}$, with frequencies equal to the difference in the various LCS energies involved, and of pulse length on the order of $c\tau_N$. Let us suppose that our detector is initially in the rth LCS when the wave-zone pulse from the emitter arrives. Then we can write the detector wave function as†

$$\psi^{(4)}(\mathbf{x}, t) = \sum_{N=1}^{\infty} c^{(4)}_N(t)\,\chi^{(4)}_N(\mathbf{x}) \exp(-iE_N t) \tag{5.3}$$

where $c^{(4)}_N(x_3/c) = \delta_{Nr}$ and $E^{(2)}_N = E^{(4)}_N \equiv E_N$.

Now, if the radiation reaction potential $A^{(4)}_{(-)}$ is very small compared to the retarded potentials $A^{(J)}_{\text{ret.}})$ $J \neq 4$ in (5.2), then the equation of motion of the detector atom, for $\tau_N + x_3/c > t > x_3/c$ can be approximated by

$$\left(-i\partial\!\!\!/ + m - e \sum_{J\neq 4=1}^{4} A^{(J)}_{\text{ret.}}\right)\psi^{(4)}(\mathbf{x}, t) \simeq 0 \tag{5.4}$$

In addition, if we use (2.17), (2.2) and (5.1) in (5.4), it can be written as

$$\left[-i\partial\!\!\!/ + m - \frac{\alpha\gamma^0}{|\mathbf{x} - \mathbf{x}_3|} + \left(\frac{e}{4\pi}\right)\frac{(\boldsymbol{\gamma}\cdot\mathbf{J}^{(2)}(t_r))}{|\mathbf{x}|}\right]\psi^{(4)}(\mathbf{x}, t) \simeq 0 \tag{5.5}$$

where

$$\mathbf{J}^{(2)}(t_r) \equiv -e\int dx^{3\prime}\,\psi^{(2)}(x', t_r)\,\boldsymbol{\gamma}\psi^{(2)}(x', t_r) \tag{5.6}$$

Inserting (5.3) into (5.5) yields the equation of motion for the coefficients $c^{(4)}_N(t)$ in (5.3) as

$$i\dot{c}^{(4)}_N(t) \simeq \sum_M {}^{(4)}\langle N|\frac{e\boldsymbol{\gamma}\cdot\mathbf{J}^{(2)}(t_r)}{4\pi|\mathbf{x}|}|M\rangle^{(4)}\, c^{(4)}_M(t)\exp[i(E_N - E_M)t] \tag{5.7}$$

which, because $\psi^{(4)}$ is localized into a volume on the order of $1/m\alpha \ll |\mathbf{x}_3|$, can be written as

$$i\dot{c}^{(4)}_N(t) \simeq \sum_M \left[\frac{ie\Delta_{NM}\,\mathbf{x}^{(4)}_{NM}\cdot\mathbf{J}^{(2)}[t - (|\mathbf{x}_3|/c)]}{4\pi|\mathbf{x}_3|}\right] c^{(4)}_M(t)\exp(i\Delta_{NM}t) \tag{5.8}$$

where we have use the property of the Dirac coulombic functions that‡

$$\int \mathrm{d}x^3\langle N|\gamma|M\rangle = i\Delta_{NM}\,\mathbf{x}_{NM} \tag{5.9}$$

$$\mathbf{x}_{NM} \equiv \langle N|\mathbf{x}|M\rangle$$

$$\Delta_{NM} \equiv E_N - E_M$$

† See footnote on p. 219.

‡ See footnote on p. 212.

Now if we assume that the emitter wave function $\psi^{(2)}$ can be written in the form†

$$\psi^{(2)}(\mathbf{x},t)=\sum_{N} c_N^{(2)}(t)\,\chi_N^{(2)}(\mathbf{x}) \tag{5.10}$$

during its emission LCT, then, using (5.9), (5.6) can be written

$$\mathbf{J}^{(2)}(t-x_3/c)=-ie\sum_{p_q} \Delta_{pq}\mathbf{x}_{pq}^{(2)}\rho_{qp}^{(2)}(t-x_3/c) \tag{5.11}$$

where

$$x_3\equiv|\mathbf{x}_3| \qquad \text{and} \qquad \rho_{qp}^{(2)}\equiv c_q^{(2)}c_p^{(2)*}$$

Now (5.11) can be related to the wavezone electric and magnetic fields of $\psi^{(2)}(\mathbf{x},t)$ defined as

$$\mathbf{E}^{(2)}(\mathbf{x}_{(3)},t)\simeq-\frac{\dot{\mathbf{J}}(t-x_3/c)}{4\pi x_{(3)}} \qquad (\hbar=c=1) \tag{5.12}$$

$$\mathbf{B}^{(2)}(\mathbf{x}_{(3)},t)=\mathbf{N}^{(3)}\times\mathbf{E}^{(2)}; \qquad N^{(3)}\equiv\mathbf{x}_3/x_3 \tag{5.13}$$

$$\mathbf{N}^{(3)}\cdot\mathbf{E}^{(2)}(\mathbf{x}_3,t)=0 \tag{5.14}$$

by the fact that during the LCT of $\psi^{(2)}$, the approximation $i\dot{\rho}_{qp}^{(2)}\simeq\Delta_{qp}\rho_{qp}^{(2)}$ is valid, because the smallness of α implies that the LCT involves slowly damped oscillations. Inserting this approximation, and (5.11), into (5.12) and defining $\sigma_{qp}^{(2)}\equiv\rho_{qp}^{(2)}\exp(-i\Delta_{qp}t)$, we have that†

$$\mathbf{J}^{(2)}(t-x_3/c)\simeq-4\pi i x_3\sum_{p_q}\mathbf{E}_{pq}^{(2)}(\mathbf{x}_{(3)},t)\exp(i\Delta_{qp}t) \tag{5.15}$$

where

$$\mathbf{E}_{pq}^{(2)}(\mathbf{x}_3,t)\equiv\frac{-e}{4\pi x_3}\mathbf{x}_{pq}^{(2)}\Delta_{pq}^2\sigma_{qp}^{(2)}(t-x_3/c) \tag{5.16}$$

Insertion of (5.16) into (5.8) yields the equation of motion of the detector atom's induced response, to the emitter wave-zone fields as

$$i\dot{c}_N^{(4)}(t)\simeq-\sum_{M\,pq}\left[\left(\frac{\Delta_{NM}}{\Delta_{pq}}\right)e\mathbf{x}_{NM}^{(4)}\cdot\mathbf{E}_{pq}^{(2)}(\mathbf{x}_3,t)\right]c_M^{(4)}(t)\exp[i(\Delta_{NM}+\Delta_{qp})t] \tag{5.17}$$

With the boundary condition that the detector atom is initially in the Mth LCS ($c_{(4)}^{N}(x^3/_c)=\delta_{Nr}$), then (5.17) becomes for $t\gtrsim x^3/_c$

$$i\dot{c}_N^{(4)}(t)\simeq-\sum_{p_q}\left[\left(\frac{\Delta_{NM}}{\Delta_{pq}}\right)e\mathbf{x}_{NM}^{(4)}\cdot E_{pq}^{(2)}(\mathbf{x}_3,t)\right]\exp[i(\Delta_{NM}+\Delta_{pq})t] \tag{5.18}$$

Since $\alpha\ll 1$, then $E_{pq}^{(2)}(\mathbf{x}_3,t)$ will be slowly varying over $\Delta t\sim 1/\Delta_{pq}$. Then (5.18) implies that the tendency of the detector atom ψ^4 to make an induced

† This substitution is valid because $i\dot{\rho}_{qp}^{(2)}\simeq\Delta_{qp}\rho_{qp}^{(2)}$, and hence $\sigma_{qp}^{(2)}$ will be a slowly varying function of time, compared to $\exp(i\Delta_{qp}t)$.

LCT from the Rth LCS to the Sth LCS (where either $R > S$ or $R < S$) is proportional to $|\mathbf{x}_{SR}^{(4)}|^2$. This is because this tendency is proportional to $|c_S^{(4)}|^2$ which is in turn proportional to $|\mathbf{x}_{SR}^{(4)}|^2$ from (5.18). It is this induced response (or induced transition) which represents the act of the detector atom as it records the measurement radiation emitted by $\psi^{(2)}$ and detected by $\psi^{(4)}$ in terms of 'photons' whose frequencies are equal to the LCS energy differences, divided by Planck's constant.

Of course, these 'induced transitions' and the previously discussed 'spontaneous transition' have been developed within the framework of a deterministic, non-linear, wave-mechanical theory of elementary measurement, and the question arises as to how this can be related to the apparently statistical results of quantum theory. A connection does exist between the predictions of this nonlinear measurement theory and those of quantum mechanics, and will be discussed in the next section, where we will derive the Planck blackbody radiation formula.

6. *Statistics, 'Hidden Variables', and a Derivation of the Planck Blackbody Radiation Formula, in Elementary Measurement Electrodynamics*

An interesting connection exists between this essentially deterministic theory, of the elementary measurement of charged, non-linear, matter fields in space-time, and the statistical predictions of conventional quantum mechanics of non-overlapping hydrogen atoms, in configuration space. First of all, as was noted previously, the LCS are in direct correspondence with the conventional eigenfunctions of Dirac hydrogen atoms. Secondly, if we consider a model universe made up of a very large number of non-overlapping hydrogen atoms [as described by (2.22)] then in order to make a deterministic prediction from the theory, we must have the required initial data needed to run the large number of associated non-linear differential equations of the system in space-time. Because of the presence of advanced and retarded non-linearities in (2.22), and because the smallness of $\alpha \lll 1$ allows us to expand these potentials in powers of $|\mathbf{x} - \mathbf{x}'|/c$, then both the initial values of $\psi^{(2K)}(\mathbf{x}, t)$ *and* their derivatives (the range of the order of which depends on the power of the expansion approximation taken) are required initial data to run the equations. The conventional quantum mechanical problem would require only the initial values of the quantum mechanical wavefunction in configuration space, in order to run the associated, linear, first-order quantum mechanical equation (which for non-overlapping atoms would correspond to specifying the initial state of each atomic wavefunction, in an overall wavefunction made up of the products of the atomic wavefunctions). Hence elementary measurement electrodynamics, applied to wave mechanics in space-time, represents a kind of 'hidden variable' theory with respect to conventional quanum mechanics. This is because a greater amount of initial data (than that required by conventional quantum mechanics) for the aggregate of atoms, in space-time, is required by the theory. This extra initial data could play

the role of 'hidden variables', even though the *raison d'être* of this theory is *not* based on the postulate that quantum mechanics is somehow incomplete, and hence requires the *ad hoc* insertion of sub-quantal degrees of freedom.† If the required initial data is unknown‡ for the large aggregate of atoms, then statistical predictions would arise, in this theory, from the application of classical statistics to the non-linear wave-mechanical microstructure. Interference effects would still be possible, within this classical statistics, because of the wave-mechanical structure upon which it was being applied.

In order to see this in more detail, let us calculate the 'probability per unit time' of a 'spontaneous transition', and an 'induced transition', for a typical atom in the aggregate, treated statistically. First of all, the calculation performed in Section 4 (generalized to an aggregate of atoms), yields the result [from (4.11)] that the 'probability'§ of a spontaneous LCT out of the Rth LCS, $P_R^{(2K)}$, is

$$1/\tau_R^{(2K)} \equiv P_R^{(2K)} \sim \sum_{L=1}^{} \frac{4\alpha}{3c^3} \Delta_{RL}^3 |\mathbf{x}_{RL}^{(2K)}|^2 \qquad (K = 1, 2, \ldots, N/2) \qquad (6.1)$$

This can be written in the form

$$P_R^{(2K)} = \sum_{L=1}^{R} A_{RL}^{(2K)}; \qquad A_{RL}^{(2K)} = \frac{4\alpha}{3c^3} \Delta_{RL}^3 |\mathbf{x}_{RL}^{(2K)}|^2 \qquad (6.2)$$

where $A_{RL}^{(2K)}$ is the 'probability' of spontaneous LCT from the Rth LCS to the Lth LCS. It is interesting to note that (6.2) is immediately recognizable as the well-known 'Einstein A coefficient' heretofore consistently derivable‖ only within the framework of quantum electrodynamics. However, if

† The concept of 'hidden variables' in quantum theory was first developed by David Bohm. For the latest work in this vein, see Bohm, D. and Bub, J. (1966). *Review of Modern Physics*, **38**, 447.

‡ In the same sense that knowledge of a 'macro-state' of an aggregate of atoms does not necessarily imply that complete knowledge of the 'micro-state' structure is known. This is because a macro-state is degenerate with respect to the possible combinations of micro-states, in the aggregate, which could give the said macro-state its apparent properties. Of course the required initial data, needed to run the large number of non-linear measurement equations, involves micro-state knowledge, in terms of the wave-mechanical atoms in the aggregate. The implication, in this model, is that the universe 'knows' this micro-information and uses it in the non-linear dynamics of the elementary measurement structure. However, 'macroscopic' observers (e.g. human observers, large machines, etc.) made up of aggregates of atoms, would have difficulty in 'extracting' this micro-information. This is because the attempt at extraction would always involve the interaction of large macroscopic aggregates of atoms, whose *change in macro-state* would be used to determine the desired *micro-information.*

§ Probability is not fundamental in this type of wave-mechanical theory in space-time. Instead, statistics arise from the application of classical statistics to the microscopic wave-mechanical structure, in spacetime, for an aggregate of atoms in interaction.

‖ Many discussions of this are in the literature. Perhaps the most readable is given by Slater, J. C. (1960). *Quantum Theory of Atomic Structure*, Vol. I, pp. 140–159. McGraw-Hill. See also Sakurai, J. (1968). *Advanced Quantum Mechanics*, pp. 38–39. Addison-Wesley.

other atoms are present in the model universe, then we must also include the effects of the

$$-e \sum_{J \neq 2K}^{N} A^{(J)}_{(\text{ret.})}$$

potentials, as well as that of the $-eA^{(2K)}_{(-)}$ potential. Generalizing the calculation done in Section 5 [and specifically (6.1)] to the case of an aggregate of atoms, then the equation of motion of the $2K$th atom (assumed initially in the Rth LCS) is given by

$$i\dot{c}_N^{(2K)}(t) \simeq -\sum_{P_q} \left(\frac{\Delta_{Nr}}{\Delta_{pq}}\right) (\mathbf{x}_{NR}^{2K}) \cdot \mathbf{E}_{pq}^{(\text{aggr.})}(\mathbf{x}_{2K-1}, t) \exp\left[i(\Delta_{NR} + \Delta_{pq})\, t\right] \quad (6.3)$$

where

$$\mathbf{E}_{pq}^{\text{aggr.}}(\mathbf{x}_{2K-1}, t) = \sum_{J \neq K}^{2N} \mathbf{E}_{pq}^{(2J)}(\mathbf{x}_{2K-1}, t) \quad (6.4)$$

is the total electric field of the aggregate of atoms ($J = 1, 2, \ldots \neq 2K$) associated with the frequency $(E_p - E_q)/\hbar \equiv \Delta_{pq}$. Integrating (6.3) over $\Delta t \sim 1/\Delta_{NR}$ (and assuming that $\mathbf{E}_{pq}^{\text{aggr.}}$ is slowly varying over this time interval, because of $\alpha \ll 1$), we have†

$$\frac{\overline{|c_N^{(2K)}(t)|^2}}{\Delta t} \simeq 4\pi\alpha |\mathbf{x}_{NR}^{2K}|^2 |\mathbf{E}_{NR}^{\text{aggr.}}|^2 (\Delta t) \cos^2 \theta_{NR} \quad (6.5)$$

However, the average of $\cos^2 \theta_{Nr}$ over 4π steradians is $\frac{1}{3}$, hence (6.5) is

$$\frac{|c_N^{(2K)}(t)|^2}{\Delta t} \simeq \frac{4\pi\alpha}{3} |\mathbf{x}_{NR}|^2 (|\mathbf{E}_{NR}^{\text{aggr.}}|^2 \Delta t) \quad (6.6)$$

In addition, since there are two possible polarization directions which contribute to $\mathbf{E}_{NR}^{\text{aggr.}}$, hence it can be written as

$$\mathbf{E}_{NR}^{\text{aggr.}} = (E_{NR}^{\text{aggr.}}(1))\, \boldsymbol{\epsilon}_1 + (E_{NR}^{\text{aggr.}}(2))\, \boldsymbol{\epsilon}_2 \quad (6.7)$$

where $\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2 = 0$ and

$$\left(\boldsymbol{\epsilon}_2{}^1 \cdot \frac{\mathbf{x}_{2n-1}}{|\mathbf{x}_{2K-1}|} = 0\right)$$

If we assume an equal intensity along the two polarization directions, on the average, then $E_{NR}^{\text{aggr.}}(1) = E_{NR}^{\text{aggr.}}(2) \equiv E_{NR}^{\text{aggr.}}(0)$. Then (6.6) can be written as

$$\frac{\overline{|c_N^{(2K)}(t)|^2}}{\Delta t} \simeq \frac{8\pi\alpha}{3} |\mathbf{x}_{NR}^{(2K)}|^2 [|E_{NR}^{\text{aggr.}}(0)|^2 \Delta t] \quad (6.8)$$

Now the measurement radiation energy density, per unit frequency, detected by the $2k$th atom during $\Delta t \sim 1/\Delta_{NR}$ is

$$\mathscr{E}_{NR_{(2K)}}^{\text{aggr.}} \equiv (E_{NR}^{\text{aggr.}}(0))^2 \Delta t = \left[\frac{(E_{NR}^{\text{aggr.}}(0))^2 + (B_{NR}^{\text{aggr.}}(0))^2}{2}\right] \Delta t \quad (6.9)$$

† This is justified by equation (5.16).

Hence (6.8) is

$$\frac{\overline{|c_N^{(2K)}(t)|^2}}{\Delta t} \simeq \left(\frac{8\pi\alpha}{3}|\mathbf{x}_{NR}^{(2K)}|^2\right)\mathscr{E}^{\text{aggr.}}_{NR_{(2K)}} \tag{6.10}$$

But since $|c_N^{(2K)}|^2/\Delta t$ is proportional to the probability per unit time of an induced LCT, from the Rth LCS to the Nth LCS ($N > R$ or $N < R$) we see that the 'Einstein B coefficient' is

$$B_{NR}^{(2K)} = \frac{8\pi\alpha}{3}|\mathbf{x}_{NR}^{(2K)}|^2 \tag{6.11}$$

which again is in agreement with that calculated from quantum electrodynamic arguments.† Now having calculated the Einstein A and B coefficients, we can proceed to assume that the aggregate of atoms is in statistical equilibrium. Then the usual derivation of the Planck energy density per unit frequency formula follows in a straightforward manner,‡ and yields for the $2K$th atomic 'detector',

$$\mathscr{E}^{\text{aggr.}}_{NR_{(2K)}} = \frac{A_{NR}^{(2K)}/B_{NR}^{(2K)}}{(B_{RN}^{(2K)}/B_{NR}^{(2K)})(\exp[\Delta_{NR}/KT]-1)} \tag{6.12}$$

which, when (6.2) and (6.11) are inserted, yields the well-known formula (here $\nu_{NR} \equiv \Delta_{NR}/2\pi\hbar$ and $\hbar$ and c are explicitly shown)

$$\mathscr{E}^{\text{aggr.}}_{NR_{(2K)}} = \frac{8\pi\nu_{NR}^2}{c^3}\left[\frac{h\nu_{NR}}{\exp[h\nu_{NR}/KT]-1}\right] \tag{6.13}$$

We emphasize that this result has been derived from a non-linear, wave-mechanical, elementary measurement theory in space-time, which is fundamentally deterministic. No second quantization of the electromagnetic fields is used, and statistics arise from the application of classical statistics to an aggregate of charged matter waves in space-time. The apparent effects of 'photons', implied in (6.13), arise essentially from the non-linear response of 'detector' atoms to 'emitter' atoms, and not to an underlying granular nature of the electromagnetic field. The measurement radiation density (6.13) is operationally defined in terms of emitter and detector atoms and is not, as in Maxwell–Lorentz theory, defined independently of the act of measurement.

7. *Conclusions*

We have shown how to generalize the paradigm of Elementary Measurement, discussed elsewhere,§ into a non-linear spinor wave-mechanical

† Refer to the literature quoted in footnote on p. 221.

‡ A simple discussion of these coefficients, and their use in deriving the blackbody radiation formula of Planck, is given in Sokolov, K., Loskutov, Y. and Ternov, I. (1966). *Quantum Mechanics*, Chap. 9. Holt, Rhinehart and Winston.

§ See Leiter (1969).

theory of atomic processes in space-time. The application of this theory to non-overlapping hydrogen atoms yielded the result that the inherent 'limit cycle solutions' of the non-linear measurement equations, corresponded to the quantum levels of conventional Dirac hydrogen. Even though these LCS were solutions to a non-linear differential equation, they have the property of being a complete set of functions over the discreet functions space of the measurement. Hence superpositions of these solutions can be used to describe general LCT solutions. The LCT have the property of describing atomic transition processes, between quantum levels, in a causal and continuous manner (reduction of the wave packet). Two kinds of LCT were discussed. For an isolated hydrogen atom, the characteristic lifetime of the LCT was shown to be *identical* to that calculated from conventional quantum mechanics and quantum electrodynamics for hydrogenic levels (i.e. equal to the inverse of the sum of the relevant 'Einstein A coefficients', or on the order of 10^{-8} sec). For a hydrogen atom under the influence of the measurement radiation fields of the other hydrogen atoms in the system, in addition to the 'spontaneous transitions' described above, 'induced limit cycle transitions' are generated. These induced transitions correspond to the effective absorption or emission of a 'photon' of measurement radiation energy and have a characteristic lifetime related to the associated 'Einstein B coefficient' or on the order of 10^{-15} sec.

'Photons', in this theory, are not elementary particles (i.e. quanta of the quantized electromagnetic field), but are instead produced by the secondary dynamical effect related to the inherent nonlinear structure of the process of 'measurement radiation'.

This type of atomic theory is quite different from the conventional, linear, quantum wave-mechanical theory in configuration space. This is because it is not inherently statistical in nature. Instead it represents a deterministic theory of charged matter waves, interacting in space-time through an elementary measurement electrodynamical coupling. Even though there is no strict 'uncertainty principle' in this theory, the Schwartz Inequality,† inherent in the Dirac wave-mechanical structure of this model, implies that a mathematical equivalent of the Heisenberg Uncertainty Principle may be deduced. This equivalent implies the same physical constraints, on atomic systems, as that of the uncertainty principle in conventional quantum theory. However, the lack of a wave-particle duality, in this theory, implies that 'probability' and 'uncertainty' are not fundamental to its interpretation. Statistics arise from the application of classical statistical mechanics to the wave-mechanical micro-structure, for an aggregate of hydrogen atoms in interaction.‡ Hence, even though the electromagnetic fields are not second quantized, in the theory, both induced

† See footnote on p. 208. For example $\Delta P^{(2K)} \Delta x^{(2K)} \geqslant \hbar; K = 1, 2, \ldots, N/2$ would be valid, where $\Delta P^{(2K)}$ and $\Delta x^{(2K)}$ are the root-mean-square values of the operators $\hat{\mathbf{P}} = -i\nabla$ and $\mathbf{x} = \mathbf{x}$ (which appear in a natural fashion in the Lagrangian and total energy of the system).

‡ See footnote on p. 226.

and spontaneous transition 'probabilities' per unit time can be calculated, and are identical to those calculated for conventional quantum electrodynamics. Hence a consistent derivation of the Planck blackbody radiation formula can be given, in which the associated electromagnetic field is a c-number and is not second quantized.†

It is not the purpose of this work to argue that conventional quantum mechanics and quantum electrodynamics are 'wrong'. Instead, we merely wish to demonstrate the fact that many of the predictions of quantum mechanics can be reproduced within the framework of Elementary Measurement Electrodynamics, applied to Dirac wave mechanics. Much further work needs to be done in order to understand the extent to which this theory can duplicate the main results of quantum electrodynamics,‡ and to what extent it differs.§ The fact that the Einstein A and B coefficients and the Planck blackbody formula can be derived *without* quantizing the field presents a critical precedent which is not trivial in its implication.

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† See Section 5 of this article.

‡ We speculate that most quantum processes, which can be accounted for within the framework of Semi-classical Quantum Theory, can also be calculated within the framework of this new theory. For a general background on semi-classical radiation calculations see the reference quoted below. In addition, since the probability of an induced limit cycle transition for an atom (in an aggregate of atoms) is given by equation (6.7), we see that the probability distribution for the emission or absorption of a 'photon' is proportional to the square of the total electric field acting on the atom. This result is in agreement with quantum electrodynamics, and implies the usual interference effects, in terms of probabilities, that one would expect from a second quantized formalism. The details of these calculations are straightforward and will be the subject of a future publication. Finally, the manner in which this theory may account for the 'radiative correction' type of process will be investigated following the suggestions made in footnotes on pp. 208 and 217. For related calculations, see Jaynes, E. T. (1969). *Physical Review*, **179**, No. 5, 1253.

§ If a theory like this plays the role of a 'covering theory', relative to conventional quantum mechanics, then one area in which the two theories might give different results involves the measurement of correlations. In particular the study of the polarization correlation of pairs of optical (or high-energy) photons in 'Einstein–Podolsky–Rosen' types of experiments. For the study of a recently proposed experiment involving optical photons, see Shimony, A. *et al.* (1969). Proposed Experiment To Test Local Hidden Variable Theories, *Physical Review Letters, 23*, 15, 880, (1969). See also Jaynes, E. and Stroud, C., *Phys. Review 1*, 106, (1970). For a discussion about the properties of a semiclassical radiation theory which (although different from our theory) has a similar behavior with regard to predictions about spontaneous limit cycle transitions. For this

reason, the experiment suggested by Jaynes and Stroud, on pages 118–119 of their paper, would also be a test of our theory. In particular, the lifetimes of excited atomic states should be distorted by the "strong excitation" of extremely intense laser beams, in the experiment they suggest. This also implies that for *extremely* high temperatures the distortion of the "effective Einstein A coefficient" will cause the blackbody radiation spectrum to be itself distorted. The details of this will be forthcoming in a future publication.